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FLOW STRESS-STRAIN RELATIONSHIPS IN TENSION TESTS OF STEEL

TECHNICAL REPORT WAL TR 834.2/10

BY

JOHN NUNES

DATE OF ISSUE - APRIL 1963

WATERTOWN ARSENAL
WATERTOWN 72, MASS.

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Plastic flow

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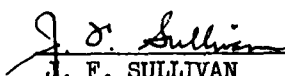
FLOW STRESS-STRAIN RELATIONSHIPS IN TENSION TESTS OF STEEL

ABSTRACT

From the observation that the tensile flow stress curve remains reasonably linear once local necking occurs, a useful relationship between the uncorrected flow stress curve and the corrected flow stress curve is derived utilizing Bridgman's correction formula. The slopes of these curves, m_t (corrected flow stress) and m (uncorrected flow stress), are linear strain-hardening indices. They are analytically and experimentally shown to be related by a constant factor which appears to be 0.5. It is possible to approximate the corrected flow stress curve and the linear strain hardening factor m_t employing the relationships proposed here. It is further shown that a useful linear strain hardening index can be evaluated at a necking strain of 1.0 using the relationships of $m = 0.4\sigma_1$ (the uncorrected flow stress at a necking strain of 1.0).

JOHN NUNES
Materials Engineer

APPROVED:

J. F. SULLIVAN
Director
Watertown Arsenal Laboratories

INTRODUCTION

Most of the past work done on tensile deformation has been concentrated on the uniform strain region of the tensile flow stress curve where complex stress systems introduced by localized deformation (necking). are. not present. However, it may be possible to obtain some useful true stress-true strain relations in the necking region provided a few simple assumptions are made concerning maximum load and the start of necking, and that Bridgman's flow stress correction factor is used.[1]

An attempt will be made here to determine and evaluate such a function and to show that the complete flow stress tensile curve to fracture can be analytically treated.

ANALYSIS

In an analysis of a simple tensile flow stress curve, some basic assumptions must be made about the material tested which are: (1) it is polycrystalline and isotropic, (2) no metallurgical transformations occur, and (3) there is no change in deformation mechanism occurring during the test. These factors can act to change the shape of the tensile curve, and hence would influence any functional relationships established between true stress and true strain. With these assumptions in mind, Figure 1 has been constructed and represents the typical tensile curves obtained both for the specimen load area and the flow stress-strain parameters. Referring to the peak load shown on the load-area curve, it is assumed that necking generally starts at the point where $\frac{dL}{dA} = 0$. Also the end of the uniform strain (the

Figure I. TENSILE FLOW CURVES SCHEMATIC

strain at maximum load, ϵ_{ml}) is observed to equal the strain hardening exponent, n, from the power law equation,

$$\sigma = S_1 \epsilon^n \qquad \ldots \ldots (1)$$

and hence $\frac{d\sigma}{d\epsilon} = \sigma$. Although this power law equation has had no theoretical or analytical justification yet, it has proven to uniquely describe most

-3-

true stress and true strain data, particularly when the other variable factors previously referred to have been controlled or taken into consideration.

Although the emphasis here will be on the relationships observed between the measured true stress (σ) and the corrected true stress (σ_t), it will be useful to review some basic definitions and derivations. First are the definitions of true stress (σ) and true strain (ϵ), where

$$\sigma = \frac{L}{A} \quad \text{and} \quad \epsilon = \ln \frac{A_o}{A}$$

$$L = \text{load}$$

$$A = \text{instantaneous area}$$

$$A_o = \text{original area}.$$

The true strain can also be derived from the concept of constancy of volume which postulates that the volume of deformed material during plastic flow remains constant as follows:

$$\text{volume} = lA$$

$$\frac{dl}{l} = \frac{-dA}{A} = d\epsilon$$

$$\text{where } l = \text{length}.$$

However, this also involves a basic definition, which is that the change in length or displacement over a given gage length is equal to the strain. Referring back to the definitions of true stress, the following relationships between the tensile flow stress and the load-area curves can be obtained

$$\frac{d\sigma}{d\epsilon} = \sigma - \frac{dL}{dA} \qquad \qquad \dots \dots (2)$$

when

$$\frac{dL}{dA} = 0, \quad \frac{d\sigma}{d\epsilon} = \sigma.$$

It can readily be seen that the above relationships at maximum load (dL=0) go to a form which is compatible with the empirically postulated power law equation 1 at maximum load.

Although equation 2 is valid for the entire flow stress-strain curve, equation 1 is not (once necking starts), due to the highly localized strain resulting at the minimum cross-section of the neck. This is also accompanied by an induced hydrostatic stress component which

acts to increase the measured flow stress values at this region. By employing Bridgman's formula, a correction for the increased flow stress (σ) based upon the neck geometry can be obtained.

$$\sigma_t = \frac{\sigma}{(1 + \frac{2R}{a})\ \ell n\ (1 + \frac{a}{2R})} \qquad \cdots \cdots (3)$$

where

a = minimum radius

R = radius of curvature of the necked region

Such corrections do not necessarily prove the validity of equation 1 as no consideration is given to the effects of strain. However, it has been suggested that the correction factor of $\frac{a}{R}$ can be replaced by the necking strain, ϵ_n,[2,3] (which is equal to the total strain, ϵ, minus the strain at maximum load, ϵ_{ml}), Previous work by Marshall and Shaw[4] resulted in data which correlated with this relationship quite well. This implies that for an untapered tensile specimen

$$\frac{a}{R} = k\epsilon_n$$

and k is approximately 1. To illustrate the validity of this expression a test was conducted on an AISI 4140 steel where the neck profiles could be determined by em-

Figure 2. a/R RATIO VERSUS NECKING STRAIN, ϵ_n FOR AISI 4140 STEEL

ploying a test system previously developed.[5] These results, which show k equal to 1, are shown in Figure 2 where the necking strain is plotted versus the a/R ratio. With this type of relation, it is now possible to obtain Bridgman's correction as a function of stress and necking strain.

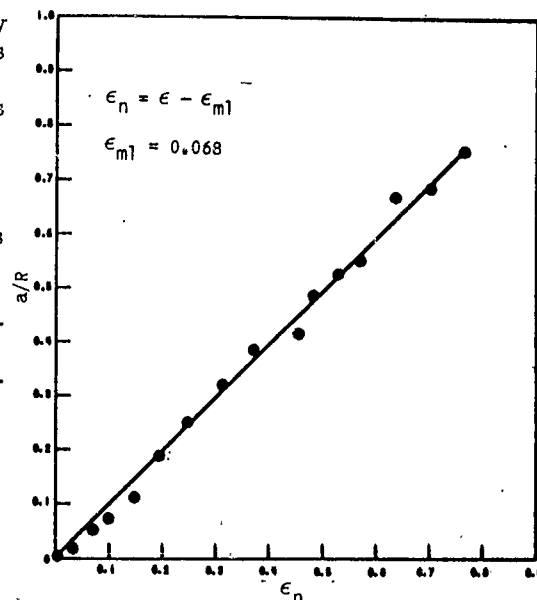$$\sigma_t = \frac{\sigma}{(1 + \frac{2}{\epsilon n})\ \ell n\ (1 + \frac{\epsilon n}{2})} \qquad \cdots \cdots (4)$$

Also from this, a simple table can be calculated which gives the correction value for a given necking strain. This was done and is tabulated in Table I.

-5-

TABLE I

BRIDGMAN'S FLOW STRESS CORRECTION FACTOR
BASED ON THE NECKING STRAIN, ϵ_n (where $a/R = \epsilon_n$)

ϵ_n	0	0.010	0.020	0.030	0.040	0.050	0.060	0.070	0.080	0.090
0.000	1.000	1.003	1.005	1.008	1.010	1.012	1.015	1.017	1.020	1.022
0.100	1.025	1.027	1.030	1.032	1.034	1.036	1.040	1.042	1.044	1.046
0.200	1.048	1.051	1.053	1.055	1.058	1.060	1.062	1.065	1.067	1.069
0.300	1.072	1.074	1.076	1.078	1.080	1.083	1.085	1.087	1.089	1.092
0.400	1.094	1.096	1.098	1.100	1.103	1.106	1.108	1.110	1.112	1.114
0.500	1.116	1.118	1.120	1.122	1.124	1.126	1.128	1.131	1.133	1.135
0.600	1.137	1.139	1.141	1.143	1.145	1.147	1.149	1.151	1.153	1.155
0.700	1.158	1.160	1.162	1.164	1.166	1.168	1.170	1.172	1.174	1.176
0.800	1.178	1.180	1.182	1.184	1.186	1.188	1.190	1.192	1.194	1.196
0.900	1.197	1.199	1.201	1.203	1.205	1.207	1.209	1.211	1.213	1.215
1.000	1.216	1.218	1.220	1.222	1.224	1.226	1.228	1.230	1.231	1.233
1.100	1.235	1.237	1.239	1.241	1.242	1.244	1.246	1.248	1.250	1.252
1.200	1.253	1.255	1.257	1.259	1.260	1.262	1.264	1.266	1.268	1.269
1.300	1.271	-	-	-	-	-	-	-	-	-

Corrected flow stress $= \sigma_t$ $a =$ instantaneous radius at the neck
Flow stress (measured) $= \sigma$ $R =$ radius of curvature at the neck
Correction factor $= u$

$$u = (1 + \frac{2}{\epsilon_n}) \, \ell n \, (1 + \frac{\epsilon_n}{2}) \quad \text{where } \epsilon_n = a/R$$

$$u \, \sigma_t = \sigma$$

It is now possible to go into the next step, which is to show the relationship between the corrected and uncorrected flow stress equations.

Illustrated in Figure 3 are the corrected and uncorrected flow stress curves for the 4140 steel, where it can be seen that at some small strain beyond necking to the vicinity of fracture these curves are reasonably linear. This has been a generally observed phenomenon for heat-treated steels at test temperatures of 200 C to temperatures of -196 C.[3,6,7,8,9] Furthermore, the work done by Bridgman on many different steels[10] to true strains as high as four (greater than 98 percent reduction of area) has shown these curves to be linear for both the measured and the corrected flow stress parameters. This indicates that a reasonable assumption of linear strain-hardening also can be made once localized deformation (necking) ensues.

Returning to equation 4 and differentiating, the following relation-ship between the flow stress parameters σ and σ_t results

$$\frac{d\sigma_t}{d\epsilon} = \frac{1}{u}\left[\frac{d\sigma}{d\epsilon} - \frac{\sigma}{e_n u}\left(1 - \frac{2u}{2+e_n}\right)\right] \qquad \ldots \ldots (5)$$

where

$$u = \left(1 + \frac{2}{e_n}\right)\ ln\ \left(1 + \frac{e_n}{2}\right).$$

Also, if we assume that $d\sigma/d\epsilon$ approximates a constant, m, from the pre-vious observations shown for steels, and that $d\sigma_t/d\epsilon$ is also approximated by some constant, m_t, then the ratio of these will be some constant k as shown below:

$$k = \frac{m_t}{m} = \frac{1}{u}\left[1 - \frac{\sigma}{u e_n m}\left(1 - \frac{2u}{2+e_n}\right)\right] \qquad \ldots \ldots (6)$$

Analysis of equation 6 based upon the 4140 data shows that at any specific value of e_n up to frac-ture, k is approximately equal to 0.5. Calculated specific values which were obtained for the ex-perimental data points are plot-ted in Figure 4 and show that this approximation is quite rea-sonable. That this constant k of 0.5 could be the same for all steels is an interesting possibil-ity which could be rationalized on the basis that the neck geom-etry primarily controlled the strain hardening curve and was the same at any given necking strain. Experimental confirmation of this is shown in Figure 5 where the slope values as determined by Bridgman[10] are plotted. The re-sulting k value, obtained with a high degree of reliability due to the small amount of scatter present, is 0.52. These particular data did not intercept at the zero coordinate due, most likely, to the test specimen geometry which was tapered and of a gage length 3 times the diameter. These factors result in a higher than normal flow stress curve, as it may be considered equivalent to prestraining a specimen under such conditions. The tests conducted on the AISI 4140 steel were run on specimens which were

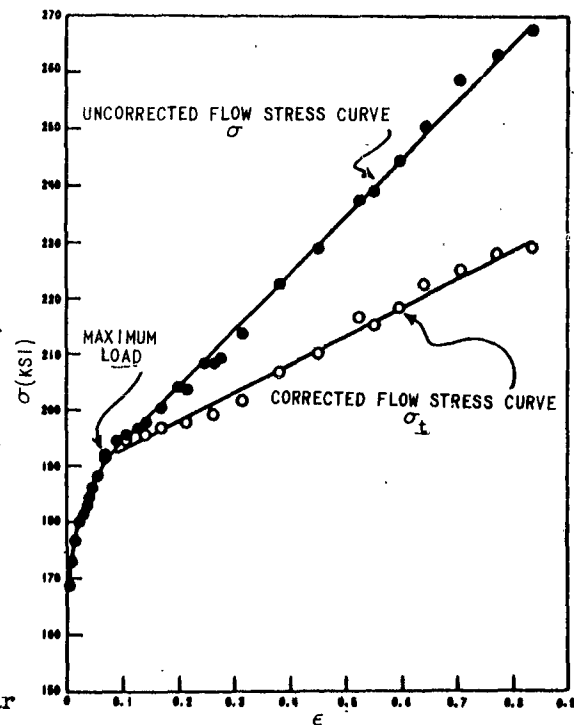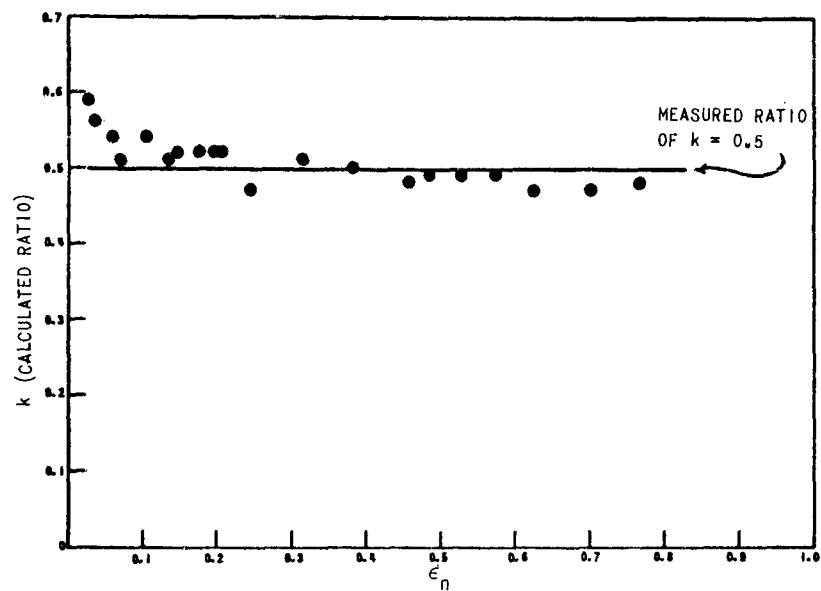Figure 3. CORRECTED AND UNCORRECTED FLOW STRESS-STRAIN CURVE FOR AISI 4140 STEEL

-7-

Figure 4. COMPARISON OF THE CALCULATED VALUES OF k
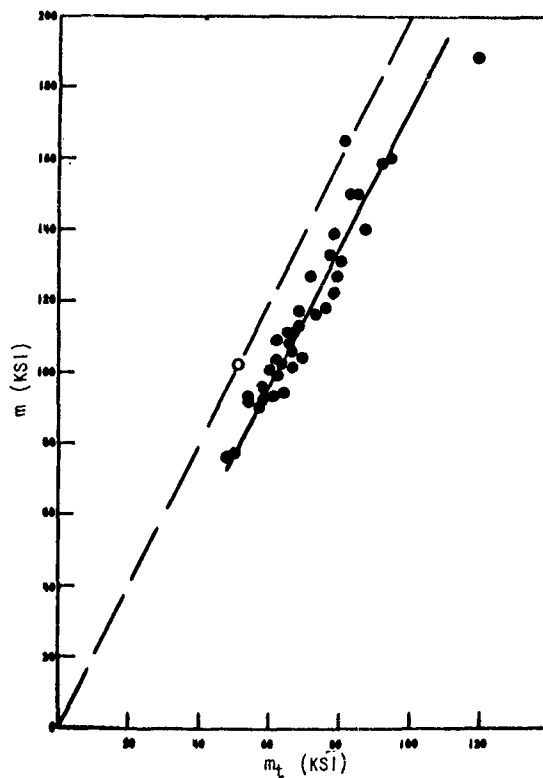VERSUS THE NECKING STRAIN FOR AISI 4140 STEEL

Figure 5. UNCORRECTED SLOPE m VERSUS
THE CORRECTED SLOPE m_t

-8-

untapered and had the minimum gage length to diameter requirements of 4.[3] This is shown as an open point in Figure 5 which falls on the postulated zero intercept slope, i.e., $m_t/m = k$. It is immediately apparent that with this relationship the complete corrected flow stress curve to fracture may be obtained easily.

Assuming that k is constant (0.5) for all steels, it is then apparent that the σ/m ratio will be a function of ϵ_n. The following equation has been obtained from equation 6 to illustrate the form of this relationship:

$$\frac{\sigma}{m} = \frac{(.5u-1)\ (2+\epsilon_n)\ u\epsilon_n}{2u\ -\ (2+\epsilon_n)} . \qquad \qquad \ldots \ldots (7)$$

The obvious relationship of a constant strain hardening slope, m, simply dependent on the flow stress at a given ϵ_n is an important factor that can be used in basic flow stress analysis. Such analysis could be analogous to the strain hardening exponent, n, for nonlinear strain hardening where the slope $d\sigma/d\epsilon = \sigma$ at a $\epsilon_n = 0$. In the linear case, an example can be chosen for $\epsilon_n = 1$ which shows that $m = 0.4\sigma_1$. With this function, analysis of flow stress behavior in the region of linear strain hardening can be readily obtained and can be further expanded to include m_t which has been shown to be simply related to m by a factor of 0.5.

SUMMARY AND CONCLUSION

Based upon the analysis and data shown here, it can be concluded that by employing Bridgman's correction factor a useful relationship between the corrected and uncorrected flow stress curves can be derived. With this type of relation, the complete flow stress curves can be corrected rather simply, provided that the basic assumption of a linear flow stress-strain curve after maximum load is made. This linearity, which can be assumed to generally hold for steels where considerable experimental evidence has been obtained, can also be utilized in an analysis of the linear strain hardening indices m and m_t which are obtainable after necking occurs. It was shown that the relationship between these slopes is relatively a constant (k) of 0.5 where $m_t/m = k$ for steels in general, based primarily upon the experimental results of Bridgman. This ratio (0.5) has also been successfully approximated from an equation derived from Bridgman's correction formula in an AISI 4140 steel tested here where

$$k = \frac{1}{u} \left[1 - \frac{\sigma}{u\epsilon_n m} \left(1 - \frac{2u}{2+\epsilon_n} \right) \right].$$

It has been further shown that for the value of $k = 0.5$ the function of σ/m is equal to some constant at any given strain, e.g., when

$\dot{\epsilon}_n = 1$, $\sigma_1/m = 2.5$ or $m = 0.4\sigma_1$. Although it has not been proven, it can be assumed that this functional relationship of σ/m at constant strains remains constant due to the constant relationship of m_t to m. However, further experimental proof is necessary to verify this assumption.

REFERENCES

1. BRIDGMAN, P. W., The Stress Distribution at the Neck of a Tension Specimen, Transactions, ASM, v. 32, 1944, p. 553-574.

2. LARSON, F. R., and NUNES, J., Low Temperature Plastic Flow and Fracture Tension Properties of Heat-Treated SAE 4340 Steel, Watertown Arsenal Laboratories, WAL TR 834.2/1, December 1959; also Transactions, ASM, v. 53, 1961, p. 663-682.

3. KULA, E. B., and LARSON, F. R., Ductility Relationships in Tensile Testing, Watertown Arsenal Laboratories, WAL TR 111/25, October 1957.

4. MARSHALL, E. R., and SHAW, M. C., The Determination of Flow Stress from a Tensile Specimen, Transactions, ASM, v. 44, 1952, p. 705.

5. NUNES, J., and LARSON, F. R., A Method for Determining the Plastic Flow Properties of Sheet and Round Tension Test Specimens, Watertown Arsenal Laboratories, WAL TR 111.1/1, March 1961; also Proceedings, ASTM, v. 61, 1961, p. 1349-1361.

6. MacGREGOR, C. W., Relations Between Stress and Reduction in Areas for Tensile Tests of Metals, Transactions, AIME, v. 124, 1937, p. 208.

7. GENSAMER, M., PEARSALL, E. B., and SMITH, G. V., The Mechanical Properties of the Isothermal Decomposition Products of Austenite, Transactions, ASM, v. 28, 1940, p. 380.

8. HOLLOMAN, J. H., Effect of Heat Treatment and Carbon Content on the Work Hardening Characteristics of Several Steels, Transactions, ASM, v. 32, 1944, p. 123.

9. LARSON, F. R., and NUNES, J., Strain Hardening Properties of High-Strength Sheet Materials, Watertown Arsenal Laboratories, WAL TR 834.2/2, March 1961; also Transactions, ASM, v. 43, 1961, p. 481-503.

10. BRIDGMAN, P. W., Large Plastic Flow and Fracture, Metallurgy and Metallurgical Engineering Series, McGraw-Hill Book Company, 1952.

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